

NUMERICAL MODELLING OF ICE JAM RESISTANCE TO MAIN CHANNEL FLOW

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SUMMARY

In northern countries, subfreezing temperatures during the winter season result in the formation of ice covers on most rivers. Towards the end of the winter season, during the spring break-up period, stationary ice covers become weak in strength and break up. The resulting broken ice pieces or ice floes are significantly larger in thickness and have a rougher undersurface relative to sheet ice and impose higher hydraulic resistance. The downstream movement of the ice floes may be arrested under conditions such as an intact ice cover, bridge piers or channel constrictions, among others, thereby initiating a break-up ice jam. These ice jams most often have been observed to cause very high water stages. Detrimental effects caused by these high water levels encompass those of operational and design-related problems such as the flooding of communities due to ice-jam-induced backwater, flood risk assessments, altering of the open water flow regime, bed scour and flooding of bridges.

The ability to predict the influence of an ice jam on the main flow is of considerable importance in river engineering and can be viewed upon by its effects on the variation in the water surface levels. All other information is dependent on the foregoing. The ice jam influence on the main flow can be regarded with respect to local and global standpoints.

The primary objective of this study is to formulate the influence of the ice jam on the main channel flow. The formulation is then coupled with a two-dimensional numerical model for the simulation of the water flow regime. The data from different laboratory experiments on ice jams are reproduced numerically. Various simulations are then carried out to compute the water surface levels and velocities in channels under ice jam conditions. The numerical results are then compared with the laboratory data.

Results show that the mathematical formulation developed to predict the water surface levels and velocities along the ice jam length as well as upstream and downstream of its leading and trailing edges respectively gives satisfactory predictions.

KEY WORDS: ice jams; modelling; mathematical development; water surface profiles; flood levels; channel flow

DEFINITION OF THE PROBLEM

During late winter and early spring in Canada, ice covers which exist in rivers become weak in strength and break up. The resulting ice floes, which move downstream at high concentration, may jam at favourable locations. These locations vary widely and include river channel bottlenecks, bridge piers, curved channels, narrowing of channel widths and other hydraulic structures. Once an ice jam is initiated, the incoming ice floes accumulate at the leading edge of the jam and contribute to increasing the stresses applied on the jam. This results in the thickening of the jam by shoving and telescoping. The increase in the ice jam thickness may result in its lodging to the river banks and even in its partial or total grounding. This can be viewed as if the ice jam is anchored to the ground. Whether lodged or grounded, the ice jam gains strength by being anchored in place and results in applying more resistance to the main channel by being able to support more loads. At one point in time and in this case the ice jam in its anchored region becomes non-responsive to changes in the river hydrodynamics. These

phenomena encompassing the hydrodynamic conditions dominant during the lodgement of an ice jam and the resistance of the jam itself to the main flow expressed in terms of the spatial variation in the water surface involve many parameters that are difficult to measure and isolate in the laboratory as well as on site.

The scarcity of field and laboratory data on ice jams makes the development of a numerical model for that purpose a difficult task to achieve. Additional understanding and empirical relations are required from laboratory and field studies in order to formulate the resistance of the ice jam to the main flow. Lately a laboratory study carried out by Saade *et al.*¹ makes this possible. The findings from this laboratory study are used to formulate the ice jam resistance to the main channel flow. The formulation is then coupled with a two-dimensional hydrodynamic model.

The primary objective of this paper is to present a model developed for the simulation of ice jam resistance to the main channel flow. The model was used to reproduce six experimental test cases where an ice jam was formed and a significant drop in the water level occurred across its leading and trailing edges. Computed and measured data were compared. Overall, the model predictions compare well with measurements.

LITERATURE REVIEW

Most ice jams research has concentrated on investigating the steady state equilibrium condition for the ice jam formation processes.^{2,3} A few researchers have made an attempt to investigate surge characteristics due to ice jam release. Joliffe and Gerard⁴ reported the results of laboratory and numerical studies performed to investigate whether the presence of ice modified surge characteristics and to assess the effects of jam configuration and stream slope and resistance. Wong *et al.*⁵ conducted laboratory tests to investigate the unsteady flow condition after the release of an ice jam. The experiments were used to test an earlier model proposed by Beltaos and Krishnappan.⁶ For initial conditions the water surface profile was taken to vary linearly along the length of the ice jam. Parkinson and Holder⁷ studied ice jam development, release and surge propagation on the Mackenzie River at Norman Wells during the 1982 spring break-up. Rivard *et al.*⁸ presented the data and analysis of the water level profile through a long ice jam on a reach of the Mackenzie River. Gogus and Tatinleaux⁹ studied the mean characteristics of asymmetric flows below ice jams. A study by Ferrick *et al.*¹⁰ has been ongoing on the interaction between a surge and the intact ice cover. The work is based on field observations performed during controlled flow releases from river dams so as to break and clear the ice cover before the spring run-off arrives.

Previous studies did not fully address the ice jam resistance to the flow in terms of the rise in the upstream water level and the phreatic water level variation along the ice jam length and did not consider ice jams that are lodged in place. It is also obvious from the previous research works that field and laboratory data on ice jams are limited.

In this study the formulation for ice jam resistance to the main channel flow and its coupling with a two-dimensional mathematical model are presented. The mathematical model includes an empirical coefficient in the ice resistance terms in order to allow the estimation of the backwater profile and better understand the variation in the water levels along ice jams in relation to its thickness profile. This could also be considered as a forward step towards providing information by which the maximum possible data could be extracted from available incomplete field measurements.

MATHEMATICAL MODEL

In this section the mathematical framework of the numerical model developed for the purpose of predicting the ice jam resistance to the main channel flow is presented. The model chosen has the

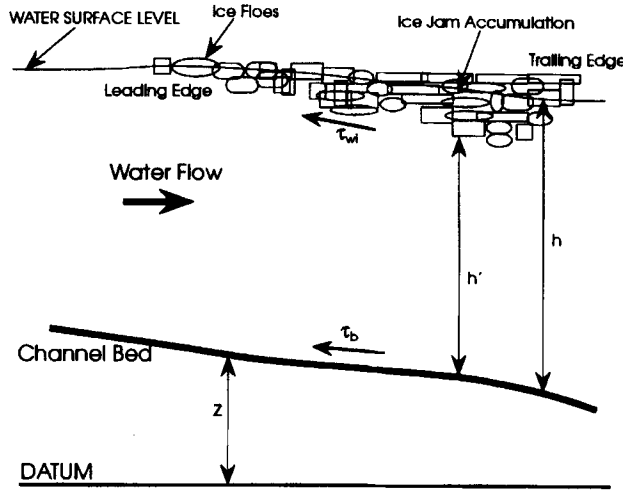


Figure 1. Definition sketch for water and ice flow

capability of evaluating the hydrodynamic conditions of the flow regime for unconfined (upstream and downstream of the leading and trailing edges of the ice jam) as well as confined (underneath the ice jam) conditions.

The hydrodynamic equations for gradually varying flow are well-established. The two-dimensional unsteady, depth-averaged continuity and momentum equations that govern the flow regime are modified to account for ice jam effects and are given as follows (Figure 1):

continuity

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(uh' + u_i C_i \pi_1 \theta) + \frac{\partial}{\partial y}(vh' + v_i C_i \pi_1 \theta) = 0, \quad (1)$$

momentum in x-direction

$$\frac{\partial}{\partial t} uh' + \frac{\partial}{\partial x} u^2 h' + \frac{\partial}{\partial y} uvh' = \frac{\tau_{(wa)x}}{\rho} - \frac{\tau_{(wi)x}}{\rho} + P_x, \quad (2)$$

momentum in y-direction

$$\frac{\partial}{\partial t} vh' + \frac{\partial}{\partial x} uvh' + \frac{\partial}{\partial y} v^2 h' = \frac{\tau_{(wa)y}}{\rho} - \frac{\tau_{(wi)y}}{\rho} + P_y, \quad (3)$$

where $h' = h - \pi_1 \theta$, h is the depth of water up to the phreatic water surface, t is the time, u and v are the velocities in the x - and y -directions respectively, θ is the ice cover thickness, $\tau_{(wa)x}$ and $\tau_{(wa)y}$ are the shear stresses at the water-air interface in the x - and y -directions respectively, C_i is the surface area ice concentration, S_i is the specific gravity of ice, π_1 is an empirical constant describing the water level within an ice jam, ρ is the water density and P_x and P_y are the water and ice pressure force components. The shear stresses at the water-ice interface are given by the following expressions in the x - and y -directions respectively:

$$\tau_{(wi)x} = \rho C_d^{wi} \sqrt{[(u - u_i)^2 + (v - v_i)^2]}(u - u_i), \quad (4)$$

$$\tau_{(wi)y} = \rho C_d^{wi} \sqrt{[(u - u_i)^2 + (v - v_i)^2]}(v - v_i), \quad (5)$$

where C_d^{wi} is the water drag coefficient on the ice underside.

A study carried out by Saade *et al.*¹ provided information concerning the spatial variation in the water levels in the case of a break-up ice jam which is lodged in place. Two π -terms given by equations (6) and (7) which represent the local penetration of the phreatic water level into the ice jam from its underside at point i along the ice jam length have been identified. The π -term given by equation (7) evaluates the spatial distance along the length of the ice jam as a function of its total length, x_i/L_j . The π -term given by equation (6) provides some evidence as to the development of the phreatic water level and ice jam thickness as a function of its position along the ice jam length.

$$\pi_{1i} = \frac{z_i^{wsel} - z_i^{bi}}{\theta_i}, \quad (6)$$

$$\pi_{2i} = \frac{x_i}{L_j}, \quad (7)$$

where i is the spatial index along the length of the ice jam evaluated with respect to the position of the leading edge, z_i^{wsel} and z_i^{bi} are the elevations of the phreatic water surface and of the bottom of the ice jam respectively, x_i is the distance along the downstream direction of the ice jam measured from the leading edge and L_j is the total length of the ice jam.

FORMULATION OF ICE JAM RESISTANCE

The ice pressure force components acting on the main flow can be determined by considering the illustration presented in Figure 2. Before proceeding with the derivation of the equations describing the ice pressure force components acting on the flow, it is necessary to understand the physical processes occurring in a typical ice jam set-up. Three zones are identified where the water undergoes major changes in its flow regime. Upstream of the leading edge of the ice jam the water flows under free surface conditions. Towards the leading edge the effect of the ice jam increases gradually. In the leading edge region the water undergoes a transition from an unconfined to a confined flow regime. Underneath the ice jam the water flow is subjected to additional resistance coming from the underside of the ice jam. Then the flow conditions, namely the flow depth and velocity, vary towards the trailing edge proportionally to the change in the ice jam thickness. At the trailing edge the water again undergoes a transition from a confined to an unconfined flow regime and the resistance of the ice jam thickness which was at a maximum at the trailing edge is suddenly lifted. Therefore the flow is expected to sharply increase in depth and decrease in velocity.

The transition of the flow regime as it passes across an ice jam should be adequately described in the governing flow equations. Therefore it is necessary to formulate the influence of the ice jam conditions on the channel flow. The ice and channel bed pressure forces including the water up to point π_{1i} in the ice jam can be obtained as follows by considering Figure 2:

$$F_A = \rho_w \pi_{1i} g \theta_4 h_4', \quad (8)$$

$$F_B = \frac{1}{2} \rho_w \pi_{1i} g h_4'^2, \quad (9)$$

$$F_C = \rho_w \pi_{1i} g \theta_5 h_5', \quad (10)$$

$$F_D = \frac{1}{2} \rho_w \pi_{1i} g h_5'^2. \quad (11)$$

The horizontal component of the gravity force and that of the resisting force due to the channel bed

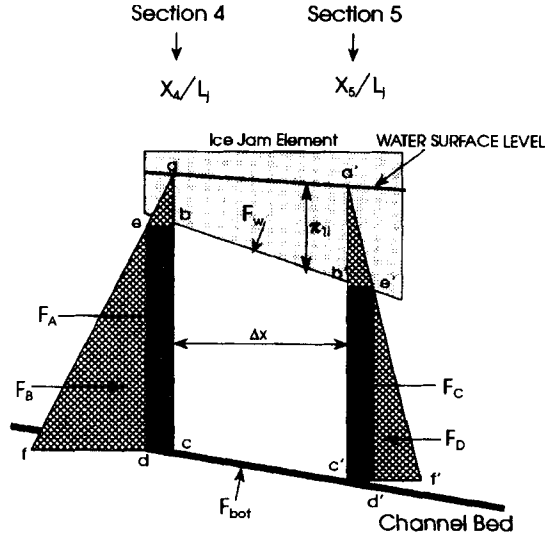


Figure 2. Forces acting on an element of water and ice

slope and that of the underside of the ice cover respectively are given as

$$F_{\text{bot}} = \rho_w g (h' + \pi_1 \theta)_{\text{ave}} (z_4 - z_5), \quad (12)$$

$$F_w = \rho_{rmw} g \pi_1 \theta_{\text{ave}} [(z_4 + h'_4) - (z_5 + h'_5)]. \quad (13)$$

Here θ is the thickness of the ice cover, h is the water depth, z is the elevation of the river bottom from datum, F_A is the hydrostatic force represented by rectangle $bode$, F_B is the hydrostatic force represented by triangle def , F_C is the hydrostatic force represented by rectangle $b'c'd'e'$, F_D is the hydrostatic force represented by triangle $d'e'f'$, F_{bot} is the pressure due to gravity acting on the element of water along the slope represented by the line joining points cc' , and F_w is the pressure force component acting along the slope of the ice cover underside and caused by the pressure force component differential represented by triangles abe and $d'b'e'$.

By considering the above forces, the following components describing the conditions dominating the change in the flow regime across an ice jam should be included in the governing water flow equations:

in the x-direction

$$P_x = g \frac{\partial}{\partial x} \left(\frac{1}{2} h'^2 + \pi_1 \theta h' \right) - g (h' + \pi_1 \theta)_{\text{ave}} S_{fx} + g \pi_1 \theta_{\text{ave}} \frac{\partial}{\partial x} (z + h'), \quad (14)$$

in the y-direction

$$P_y = g \frac{\partial}{\partial y} \left(\frac{1}{2} h'^2 + \pi_1 \theta h' \right) - g (h' + \pi_1 \theta)_{\text{ave}} S_{fy} + g \pi_1 \theta_{\text{ave}} \frac{\partial}{\partial y} (z + h'), \quad (15)$$

where

$$S_{fx} = \frac{n_b^2 u \sqrt{(u^2 + v^2)}}{R^{4/3}}, \quad (16)$$

$$S_{fy} = \frac{n_b^2 v \sqrt{(u^2 + v^2)}}{R^{4/3}} \quad (17)$$

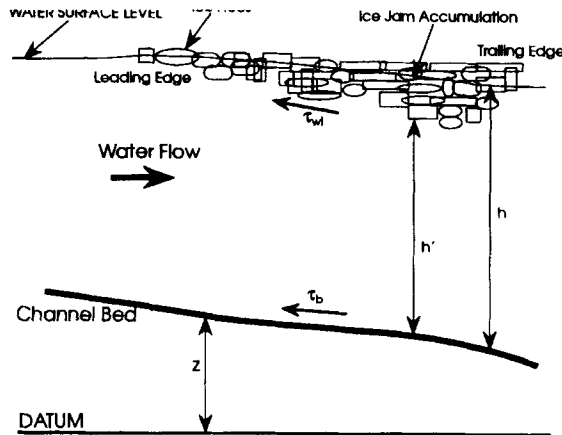


Figure 3. Finite difference grid representation

and n_b is the Manning coefficient of the river bed. It should be pointed out here that since only the bed Manning coefficient is used to account for the bed friction, the hydraulic radius R can be approximated by the flow depth h for the wide river assumption.¹¹ The justification for not using the composite Manning coefficient is traced to the fact that the water drag effect on the ice floes is taken into account in the momentum equations for water, through the terms for shear stress at the water-ice interface. These terms will also appear in the conservation of momentum of the ice accumulation.

NUMERICAL SCHEME

An explicit finite difference method based on a modified MacCormack time-splitting scheme is used to solve the depth-averaged St. Venant equations.¹² The MacCormack scheme involves the splitting of a finite difference operator into a sequence of two simpler ones, such that each split operation is divided into a predictor-corrector sequence. The two-dimensional finite difference operator $L(\delta t)$ is split into a sequence of one-dimensional operators as

$$L(\Delta t) = L_{x1}(\Delta t_{x1})L_{x2}(\Delta t_{x2})L'_{x2}(\Delta t_{x2})L'_{x1}(\Delta t_{x1}). \quad (18)$$

With reference to Figure 3, each operator consists of a predictor which discretizes the space derivatives using backward difference and a corrector which discretizes the space derivatives using forward difference. The stability condition which gives the maximum time step that can be used for this scheme is determined by the Courant-Freidrichs-Levy criterion expressed as

$$\Delta t_{\max} \leq \min\left(\frac{\Delta x_1}{u_1 + \sqrt{gh}}; \frac{\Delta x_2}{u_2 + \sqrt{gh}}\right). \quad (19)$$

INITIAL AND BOUNDARY CONDITIONS

Initial conditions

To begin with the computational process of simulating the ice jam resistance to the main channel flow, a complete set of initial conditions is required. The water velocity components are initially set throughout the computational domain at the value estimated by the ratio of discharge in the channel

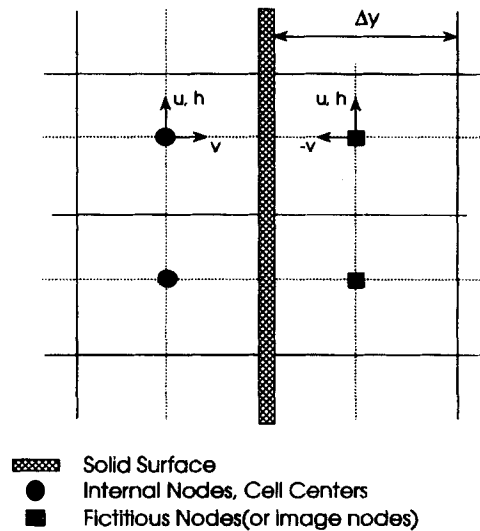


Figure 4. Specification of boundary conditions at solid surfaces

and area of flow. The ice jam thickness profile is imposed initially according to measurements. The flow depth upstream and downstream of the ice jam is set at the expected value at steady state. The flow depth underneath the ice jam is calculated initially as the difference between the elevation of the underside of the ice jam and the elevation of the channel bed.

Boundary conditions

For the hydrodynamic equations the boundary conditions for u , v and h should be specified at the solid surface and at the inlets and exits of the channel. The specification of the boundary conditions at solid surfaces such as the river banks using the method of images requires the specification of fictitious points outside the computational grid system. The normal gradient of the velocity parallel to the solid wall is set to zero and the velocity normal to the wall is set equal to the magnitude of the velocity of the inside cell, but opposite in direction as shown in Figure 4.

The water level, bottom elevation and hydraulic radius of the imaginary nodes outside the computational domain across the solid surface are taken to be the same as those immediately on the other side of the solid surface.

APPLICATION OF THE MODEL

The relation describing the penetration of the water surface into the ice jam from underneath was incorporated into the numerical model by introducing the term π_{1i} as a function of π_{2i} into the momentum equations for water. Six experimental test cases were selected for the numerical simulations, three of which were performed by Wong *et al.*⁵ and the other three by Saade *et al.*¹

Description of simulation test cases

Table I presents the conditions describing the test cases used for the numerical simulations. These tests were carried out in order to reproduce numerically the ice jam effect on the main channel flow and to verify the present numerical model computations. The six experiments were chosen to include a

Table I. Simulation test cases for ice jam resistance to flow

Numerical test case	Experimental test case	Flow rate ($\text{m}^3 \text{s}^{-1}$)	Upstream water level (m)	Downstream water level (m)	Drop in water level (%)
1	Run no. 1	0.026	0.31	0.18	42.0
2	Run no. 3	0.020	0.23	0.16	30.0
3	Run no. 5	0.025	0.21	0.19	9.5
4	6	0.0067	0.169	0.031	82.0
5	7	0.002	0.109	0.041	62
6	12*	0.0032	0.125	0.050	60

reasonable selection of the different conditions. The first three experiments were performed by Wong *et al.*⁵ in a 1.2 m wide, 20 m long rectangular flume. The ice jams were formed by feeding polyethylene blocks ($5 \times 5 \times 0.6 \text{ cm}^3$) at the upstream section of the flume and obstructing their passage with a retaining gate located about 3 m downstream. A typical ice jam thickness profile (test run 1) obtained in this experimental work is shown in Figure 5. The second three experiments were performed in a 0.297 m wide, 7.5 m long rectangular flume with glass walls. The ice jams were formed by feeding polyethylene blocks of mixed sizes or wood pieces at the upstream section of the flume and obstructing their passage by a polyethylene plate representing an ice cover. The polyethylene plate is set on a guide which allows it to move with the water surface variations while restraining it from moving along the horizontal plane. A typical ice jam thickness profile (test run 5) obtained in this experimental work is shown in Figure 6.

Numerical reproduction

Equations (6) and (7) were incorporated into the numerical model. The physical domain in all the simulation test runs was fitted into a uniform grid system. However, not the entire channel length was discretized. The channels used by Wong *et al.*⁵ and Saadé *et al.*¹ were discretized into 128×5 cells and 64×5 cells respectively. In all six test cases the cell size in the x -direction was set at 0.0625 m. In

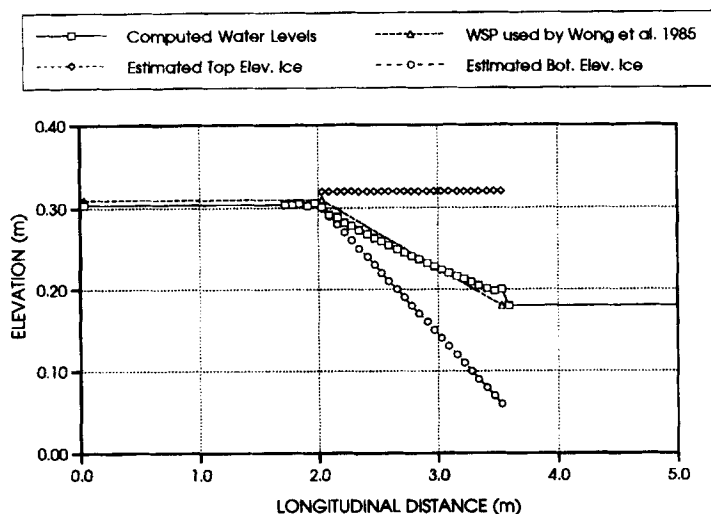


Figure 5. Water level and ice jam conditions for test case 1

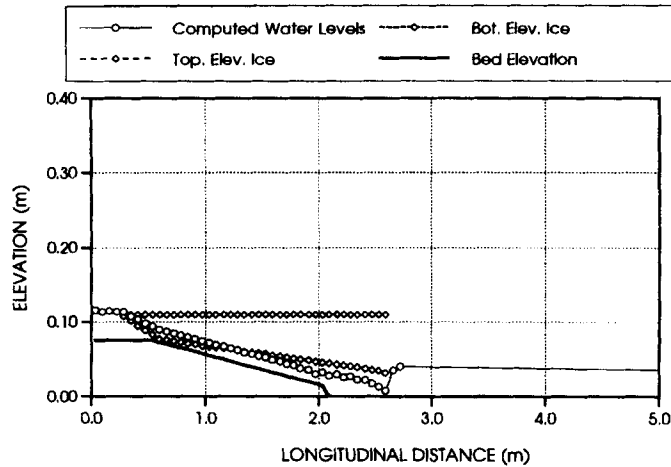


Figure 6. Water level and ice jam conditions for test case 5

the y -direction the cell size was set as 0.40 m and 0.099 m for the first three and second three tests respectively. Hence channels 8 m and 4 m in length were reproduced for test series 1 and 2 respectively. Shorter channels were used for the simulation runs in order to reduce the computation time. This could be justified by the fact that the water levels are considered only at steady state conditions.

The input to the numerical model includes the specification of two types of variables, namely those that should be exactly as measured in the laboratory and others which could be reasonably adjusted for calibration purposes. On one hand the main channel discharge and the water levels at the inlet and outlet of the channel are imposed exactly as measured. The Manning n -values of the channel bed and of the underside of the ice cover were set at 0.025 and 0.035 respectively. On the other hand the ice jam profile for the first three test runs and the Manning n -value for the ice jam underside were adjusted (within reason) in order to obtain the measured effects of the ice jam on the main flow. This is justified because the measured ice jam profiles for the first three experiments were not available. Simulation test runs with different Manning n -values for the ice cover underside ranging from 0.025 to 0.1 revealed that its effect on the upstream water level is small. Other simulations using different initial upstream water levels showed that the results at steady state remain the same. This confirms that the model presented herein predicts the rise in the water level upstream regardless of the initial conditions. The selected time step for computations was based on the CFL criterion and was small enough to ensure stability in the calculations.

Analysis and discussion of results

The primary objective of the numerical simulations is to evaluate the resistance of the ice jam to the main channel flow by computing the rise in the water level upstream of the jam. The numerical model is run until steady state conditions are attained. The water level at the most upstream location of the flume is then noted and compared with measurements.

Figure 5 illustrates the hydrodynamic and ice conditions used for the simulation test case 1. The ice jam thickness profile imposed is identified by the top and bottom elevation profiles. The ice jam thickness profile is estimated, since measurements were not available. The only available information is the trailing edge of the ice jam (at the same location as the grill) and the approximate length of the jam

(obtained from written communication with Dr. S. Beltaos, NWRI, Burlington, Ont.). Figure 5 also shows the computed and estimated water levels along the length of the channel.⁵ The estimated water surface profile is the one used for the numerical modelling carried out by Wong *et al.*⁵ in their study of ice jam releases. The profile is not measured or observed, but rather on assumption that was used as the initial condition for their release tests.

Figure 5 shows that the computed and assumed water surface profiles are in close agreement. This is not exactly what is expected; however, by looking at the problem from a fundamental point of view, a clearer interpretation of the figure could be deduced. Two aspects should be considered, namely the model calculations and the formation of the ice jam. By definition the computed flow depth using the present model is defined as the vertical distance limited by the channel bed and the underside of the ice cover at the top. Therefore it is expected that the computed water surface profile will follow the underside ice jam profile. In all of the first three simulation test cases the water surface profile does not adhere to what is expected. This can be reasoned by considering the conditions under which the ice jams have been formed in the studies performed by Wong *et al.*⁵ In these tests the jamming of the polyethylene blocks was achieved by feeding the blocks of the upstream section of the channel in a uniform flow and obstructing the passage of the blocks with the help of a porous gate located about 3 m downstream and placed along the full depth of flow. The porous gate consisted of closely spaced vertical rods that were held together by two horizontal beams. The blocks accumulated at the gate and produced jams that generally resembled natural ones, with a steep hydraulic gradient at the toe and increased stage upstream.

It is evident that the submerging ice pieces are not allowed to escape the ice jam with the flow and that the erosion of ice floes is not possible. In addition, the ice jam is supported (lodged or being held back) by the gate along its entire trailing edge frontal area perpendicular to the flow. To a certain degree this differs from what occurs in nature, since ice jams are supported by a strong downstream ice cover whose relative thickness is small compared with the ice jam thickness at the trailing edge and greatest where the ice jams are either totally or partially grounded.

Based on the above discussion, it is not surprising to note that the profile of the ice jam underside is linear and smoothly varying. Also, based on the computational results and observations from the experiments carried out in this study, the part of the ice jam below the water level shown in Figure 5 should probably be eroded under normal conditions.

The latter deductions and discussion are complemented by the results of simulation test case 5 which are presented in Figure 6. This figure gives the ice jam top and bottom surface profiles, the channel

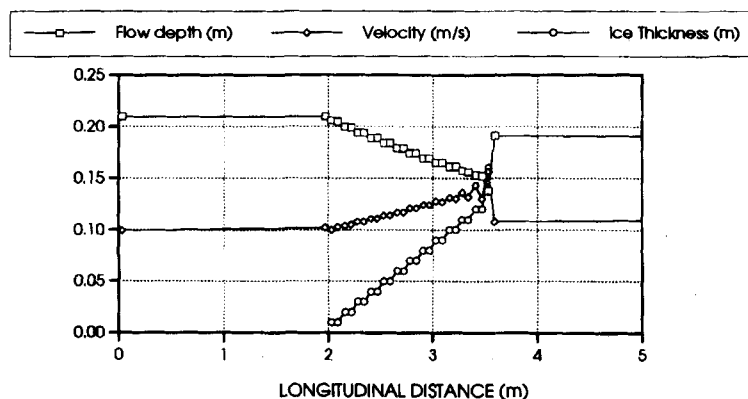


Figure 7. Flow depth and velocity profiles for a typical ice jam

Table II. Measured and computed upstream water levels

Test case	Upstream water level (measured) (m)	Upstream water level (computed) (m)	Difference (%)
1	0.31	0.304	1.9
2	0.23	0.225	2.1
3	0.21	0.209	0.5
4	0.169	0.167	1.2
5	0.109	0.114	4.6
6	0.125	0.131	4.8

conditions and the computed water surface profile. In this case the computed longitudinal flow depth profile follows closely the ice jam bottom surface profile as well as the channel bed. Adding the penetration of the water surface into the ice jam from underneath to the flow depth gives the phreatic water surface profile as measured in the laboratory experiments. The difference between the measured and computed phreatic water surface profiles would be of the same order of magnitude as that between the flow depth and ice jam bottom surface profiles, as shown in Figure 6.

In all six simulation test cases the variation in the velocity in the longitudinal direction as a function of the flow depth and ice jam thickness is consistent with the observation. As the ice jam thickness increases, the flow depth increases and the flow velocity increases as shown in Figure 7. The transition of flow is smooth at the leading edge and abrupt at the trailing edge, where a sharp increase in the water depth occurs accompanied by an equally sharp drop in the flow velocity immediately after.

Table II presents the results of the six simulation test runs. It is evident that the measured and computed ice jam resistances to the main flow, evaluated as a function of the rise in the water level at the farthest upstream location, compare well, with a maximum percentage difference of 4.8% occurring in test case 5.

CONCLUSIONS

In northern regions, ice jams may occur during the freeze-up and spring break-up periods. The increase in the ice jam size may result in major blockage of the river flow such that the water level rises upstream and drops downstream of the jam leading and trailing edges respectively. The rise of the water level upstream of the ice jam causes serious flooding of entire towns and hydraulic works situated along the river.

The unavailability of field and laboratory data on ice jamming makes it a difficult process to understand. The improved understanding of stationary ice jams that are lodged in place is an important step towards providing engineers with a better view of the process during their assessment of the hazards of ice jam failure in the design of bridges, hydro power plants, flood protection works, reservoir operations and emergency measure plans. This could be achieved through experimental and numerical modelling work.

In this paper the information obtained from ice jams that were formed in the laboratory was used in a mathematical model developed for the prediction of ice jam resistance to main channel flow. The numerical model reflects the physical phenomena of break-up ice jams and their influence on the main flow. Ice cover effects on the flow regime have been successfully accounted for in the conservation equations. Measured and computed rises in the upstream water level agree with each other very well.

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